GCF and Factoring by Grouping

The greatest common factor, or GCF, is the largest factor each term has in common. The GCF can include numbers and variables. In terms of numbers, it is the largest factor each number has in common. For example, 4 is the greatest common factor of the two numbers 4 and 20. (Notice that 2 is also a common factor; however, it is not the greatest common factor.) In terms of variables, the GCF is the largest exponent each variable has in common. For example, x^3 is the greatest common factor of x^3 and x^5. (Again, x is a common factor; however, it is not the greatest common factor).

We can factor a polynomial using the GCF (this means we are going to do a reverse distributive property). Remember distributive property means multiplication, so the reverse is division. Factoring is a form of division.

Example 1: Factor out the GCF: $15x^3 + 9x^2$

Solution: The GCF is $3x^2$ (3 is the common factor between 15 and 9 and x^2 is the common factor between x^3 and x^2. We factor out $3x^2$ from each term to get $3x^2(5x+3)$. You can check your answer by performing distributive property (you should get the original problem).

Example 2: Factor out the GCF: $12x^2y - 42xy^3 + 48x^3y$

Solution: The GCF is $6xy$. We will factor $6xy$ from each term to get $6xy(2x - 7y^2 + 8x^2)$.

Factoring by grouping is used when there is four terms in the polynomial. We will group the first two terms and factor out the GCF then group the next two terms and factor out the GCF. We will have gone from four terms to two terms and factor out what is common. Again, we will be doing reverse distributive property.

Example 3: Factor $x^2 + 3x + 4x + 12$

Solution: There are four terms; therefore, we will separate the first two terms from the next two terms and find the GCF of each pair.

$$\underbrace{x^2 + 3x}_{\text{GCF is } x} + \underbrace{4x + 12}_{\text{GCF is } 4}$$

$$\underbrace{x(x+3) + 4(x+3)}_{\text{GCF is } (x+3)}$$

$$(x+3)(x+4)$$

Example 4: Factor $3x^2 - 12 - 5x + 20$

Solution: Again, there are four terms; therefore, we will factor by grouping. Any time you factor by grouping, it is not a coincidence the two terms have the same factor in parentheses. You always want the same binomial in parentheses in the second step.

$$\underbrace{3x^2 - 12x}_{\text{GCF is } 3x} \underbrace{-5x + 20}_{\text{GCF is } -5}$$

$$\underbrace{3x(x-4) - 5(x-4)}_{\text{GCF is } (x-4)}$$

$$(x-4)(3x-5)$$

Practice Problems

Factor the Greatest Common Factor (GCF)

1. $15a + 25b$
2. $14u^2 + 35u^4$
3. $7c^3 - 28c^2d + 35cd^3$
4. $13a^3b^2 + 39a^2b - 26ab^4$
5. $4a^4b - 16a^2b^2 + 4ab$
6. $66a^3b^3 - 22ab^2 + 44a^2b^4$

Factor by grouping

7. $3x^2 + 9x + 4x + 12$
8. $2x^2 - 5x - 2x + 5$
9. $5x^2 - 30x + 2x - 12$
10. $3x^2 + 18x - 7x - 42$
11. $5x^2 + 2x + 40x + 16$
12. $6x^2 - 42x - 5x + 35$
13. $5x^2 + 40x - x - 8$
14. $5x^2 - 40x + 3x + 24$
15. $m^2 + 8mn - 3mn - 24n^2$
16. $6x^2y + 2xy^2 - 27x - 9y$